Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel International Advanced Level

Friday 9 June 2023

Afternoon (Time: 1 hour 30 minutes)

Paper reference **WMA14/01**

Mathematics

International Advanced Level
Pure Mathematics P4

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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Pearson

1. (a) Find the first 4 terms of the binomial expansion, in ascending powers of x, of

$$\left(\frac{1}{4}-\frac{1}{2}x\right)^{-\frac{3}{2}} \qquad |x|<\frac{1}{2}$$

giving each term in simplest form.

(5)

Given that

$$\left(\frac{1}{4}-\frac{1}{2}x\right)^{n}\left(\frac{1}{4}-\frac{1}{2}x\right)^{-\frac{3}{2}}=\left(\frac{1}{4}-\frac{1}{2}x\right)^{\frac{1}{2}}$$

(b) write down the value of n.

(1)

(c) Hence, or otherwise, find the first 3 terms of the binomial expansion, in ascending powers of x, of

$$\left(\frac{1}{4}-\frac{1}{2}x\right)^{\frac{1}{2}} \qquad |x|<\frac{1}{2}$$

giving each term in simplest form.

(3)

(a) We see that the given expansion is in form $(a+bx)^n$, we need to manipulate to get $(1+ax)^n$.

$$\left(\frac{1}{4}-\frac{1}{2}x\right)^{-\frac{3}{2}} = \left(\frac{1}{4}\right)^{-\frac{3}{2}}(1-2x)^{-\frac{3}{2}}$$

$$= 8(1-2x)^{-\frac{3}{2}} \quad \text{B1}$$

Get the formula for the Binomial Expansion from the formula booklet:

$$(1+x)^n = 1+nx+\frac{n(n-1)}{2!}x^2+\frac{n(n-1)(n-2)}{3!}x^3+\dots, \text{ Range of validity } |x|<1$$

Substitute into the Formula:

$$8(1-2x)^{-\frac{3}{2}} = 8\left[1+\left(-\tfrac{3}{2}\right)(-2x)+\frac{\left(-\frac{3}{2}\right)\left(-\frac{3}{2}-1\right)}{2!}(-2x)^2+\frac{\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)\left(-\frac{7}{2}\right)}{3!}(-2x)^3+\dots\right] \quad \text{M1A1}$$

$$=8\left(1+3x+\frac{15}{8}\cdot 4x^2+\left(-\frac{105}{\cancel{16}_{2}}\right)\cdot\left(-\frac{\cancel{8}^{1}}{6}x^3\right)+\dots\right)$$

$$=8\left(1+3x+\frac{15}{2}x^2+\frac{35}{2}x^3+\dots\right)$$

$$=8+24x+60x^2+140x^3+\dots \quad \text{A1A1}$$

P 7 4 3 2 9 A 0 2 3 2

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Question 1 continued

(b) Use index rule $x^a \cdot x^b = x^{a+b}$.

In this case:

$$\left(\tfrac{1}{4}-\tfrac{1}{2}x\right)^n \cdot \left(\tfrac{1}{4}-\tfrac{1}{2}x\right)^{-\frac{3}{2}} = \left(\tfrac{1}{4}-\tfrac{1}{2}x\right)^{\frac{1}{2}}$$

$$\therefore \quad n + \left(-\tfrac{3}{2}\right) = \tfrac{1}{2}$$

$$n = 2 \quad \text{B1}$$

(c)

$$\left(\tfrac{1}{4}-\tfrac{1}{2}x\right)^2 \cdot \left(\tfrac{1}{4}-\tfrac{1}{2}x\right)^{-\frac{3}{2}} = \left(\tfrac{1}{4}-\tfrac{1}{2}x\right)^{\frac{1}{2}}$$

First expand $\left(\tfrac{1}{4}-\tfrac{1}{2}x\right)^2$:

$$\left(\tfrac{1}{4}-\tfrac{1}{2}x\right)^2 = \tfrac{1}{16} - \tfrac{1}{4}x + \tfrac{1}{4}x^2. \quad \text{B1}$$

Now multiply by the expansion from part (a):

$$\left(\tfrac{1}{16} - \tfrac{1}{4}x + \tfrac{1}{4}x^2\right)\cdot\left(8 + 24x + 60x^2 + 140x^3\right)$$

$$= \tfrac{1}{16}(8) + \tfrac{1}{16}(24x) + \tfrac{1}{16}(60x^2) - \tfrac{1}{4}x\cdot(8) - \tfrac{1}{4}x\cdot(24x) + \tfrac{1}{4}x^2(8) \quad \text{M1}$$

* Remember we only need terms up to x^2.

$$= \tfrac{1}{2} + \tfrac{3}{2}x + \tfrac{15}{4}x^2 - 2x - 6x^2 + 2x^2$$

$$= \tfrac{1}{2} - \tfrac{1}{2}x - \tfrac{1}{4}x^2 + \dots \quad \text{A1}$$

Question 1 continued

P 7 4 3 2 9 A 0 4 3 2

Question 1 continued

(Total for Question 1 is 9 marks)

2.

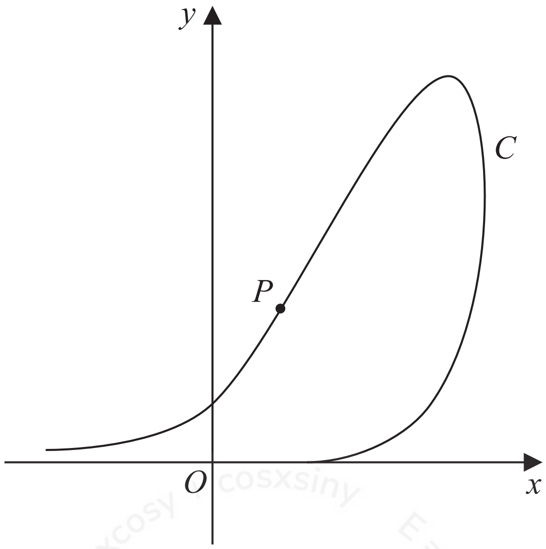

Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$2^x - 4xy + y^2 = 13 \qquad y \geqslant 0$$

The point P lies on C and has x coordinate 2

(a) Find the y coordinate of P.

(2)

(b) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y.

(5)

The tangent to C at P crosses the x-axis at the point Q.

(c) Find the x coordinate of Q, giving your answer in the form $\dfrac{a\ln 2 + b}{c\ln 2 + d}$ where a, b, c and d are integers to be found.

(3)

(a) Substitute $x=2$:

$$2^{(2)} - 4(2)y + y^2 = 13$$

Solve for y:

$$4 - 8y + y^2 = 13$$
$$y^2 - 8y - 9 = 0$$
$$(y-9)(y+1) = 0$$
$$y = 9 \qquad y = -1 \qquad \text{M1}$$

As $y \geq 0$, $y = 9$. A1

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Question 2 continued

(b) We are asked to differentiate

$2^x - 4xy + y^2 = 13$

★ Implicit differentiation:

When differentiating y, multiply by $\frac{dy}{dx}$

$\frac{d}{dx}(2^x) = 2^x \ln x$ B1

$\frac{d}{dx}(4xy) = 4y + 4x\frac{dy}{dx}$

★ Product Rule:

$\frac{d}{dx}(uv) = uv' + u'v$

$\to 2^x \ln x - 4y - 4x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$ M1A1

$2^x \ln x - 4y = 4x\frac{dy}{dx} - 2y\frac{dy}{dx}$ collect all $\frac{dy}{dx}$ on one side

$2^x \ln x - 4y = (4x - 2y)\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{2^x \ln x - 4y}{4x - 2y}$$ M1A1

(c) We got Point P in (a) $\to$ P(2, 9)

Get the gradient of tangent at P by substituting into $\frac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{2^{(2)}\ln(2) - 4(9)}{4(2) - 2(9)}$$

$$\frac{dy}{dx} = \frac{4\ln 2 - 36}{-10}$$ gradient m

Use $y - y_1 = m(x - x_1)$ to get the equation of the tangent:

$$y - 9 = \left(\frac{4\ln 2 - 36}{-10}\right)(x - 2)$$ M1

At Q, at the x-axis, $y = 0$. $\therefore$ $0 - 9 = \left(\frac{4\ln 2 - 36}{-10}\right)(x - 2)$

Rearrange to get $x = \ldots$:

$$-9 = \left(\frac{4\ln 2 - 36}{-10}\right)(x - 2)$$

$$\frac{90}{4\ln 2 - 36} = x - 2$$

$$x = \frac{\cancel{90}\ 45}{\cancel{4}\ln 2 - \cancel{36}} + 2$$
$\qquad\qquad 2 \qquad 18$

dM1 $x = \frac{45}{2\ln 2 - 18} + \frac{4\ln 2 - 36}{2\ln 2 - 18}$ $\to$ Rewrite 2 as a fraction with "$2\ln 2 - 18$" in the denominator

A1 $$\boxed{x = \frac{4\ln 2 + 9}{2\ln 2 - 18}}$$ x-coordinate in the right form.

Question 2 continued

Question 2 continued

(Total for Question 2 is 10 marks)

3. $$f(x) = \frac{8x - 5}{(2x-1)(4x-3)} \qquad x > 1$$

(a) Express f(x) in partial fractions.
(3)

(b) Hence find $\int f(x)\,dx$
(3)

(c) Use the answer to part (b) to find the value of k for which

$$\int_k^{3k} f(x)\,dx = \frac{1}{2}\ln 20$$
(5)

(a) We are asked to do Partial Fractions:

$$\frac{8x-5}{(2x-1)(4x-3)} = \underbrace{\frac{A}{(2x-1)} + \frac{B}{(4x-3)}}$$

Manipulate this to get common denominator

$$\frac{8x-5}{(2x-1)(4x-3)} = \frac{A(4x-3)+B(2x-1)}{(2x-1)(4x-3)}$$

Equate the numerators and substitute in values in order to get A, B.

$8x - 5 = A(4x-3) + B(2x-1)$

choose values to substitute in so that brackets become 0, to make your life easier!

$x = \frac{3}{4} \Rightarrow 8 \cdot \frac{3}{4} - 5 = \cancel{A(0)}^{0} + B\left(2 \cdot \frac{3}{4} - 1\right)$

$1 = \frac{1}{2}B$

$B = 2$ M1

$x = \frac{1}{2} \Rightarrow 8 \cdot \frac{1}{2} - 5 = A\left(4 \cdot \frac{1}{2} - 3\right) + \cancel{B(0)}^{0}$

$-1 = -A$

$A = 1$ A1

$$\therefore \frac{8x-5}{(2x-1)(4x-3)} = \frac{1}{(2x-1)} + \frac{2}{(4x-3)}$$ Partial Fractions

A1

Question 3 continued

(b) $\int f(x)\,dx = \int \frac{1}{(2x-1)} + \frac{2}{(4x-3)}\,dx$ M1

$= \frac{1}{2}\ln|2x-1| + 2\cdot\frac{1}{4}\ln|4x-3| + c$ A1

$\int \frac{1}{x}\,dx = \ln x + c$

Reverse chain Rule: multiply by the derivative of the bracket

$= \frac{1}{2}\ln(2x-1) + \frac{1}{2}\ln(4x-3) + c$ A1

(c) $\int_k^{3k} f(x)\,dx = \int_k^{3k} \frac{1}{(2x-1)} + \frac{2}{(4x-3)}\,dx = \frac{1}{2}\ln 20$ given value

$= \left[\frac{1}{2}\ln(2x-1) + \frac{1}{2}\ln(4x-3)\right]_k^{3k}$

$= \left[\frac{1}{2}\ln(2x-1)(4x-3)\right]_k^{3k}$ use log law $\ln a + \ln b = \ln a\cdot b$

$= \frac{1}{2}\ln(2(3k)-1)(4(3k)-3) - \frac{1}{2}\ln(2k-1)(4k-3)$ Substitute in limits

$= \frac{1}{2}\ln\frac{(6k-1)(12k-3)}{(2k-1)(4k-3)}$ M1 Expand + use log law $\ln a - \ln b = \ln\frac{a}{b}$

equate to given value → $\cancel{\frac{1}{2}}\ln 20 = \cancel{\frac{1}{2}}\ln\frac{(6k-1)(12k-3)}{(2k-1)(4k-3)}$

$20 = \frac{(6k-1)(12k-3)}{(2k-1)(4k-3)}$ dM1 Solve this

$20(2k-1)(4k-3) = (6k-1)(12k-3)$

$20(8k^2-10k+3) = (72k^2-30k+3)$

$160k^2 - 200k + 60 = 72k^2 - 30k + 3$

$88k^2 - 170k + 57 = 0$ A1 Solve quadratic

$(2k-3)(44k-19) = 0$

$k = \frac{3}{2}$ $\quad k = \frac{19}{44}$ ddM1

Since $x > 1$, only $k = \frac{3}{2}$ is the solution for k. A1

P 7 4 3 2 9 A 0 1 1 3 2

Question 3 continued

Question 3 continued

(Total for Question 3 is 11 marks)

4. Relative to a fixed origin O,

- the point A has position vector $4\mathbf{i} + 8\mathbf{j} + \mathbf{k}$
- the point B has position vector $5\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}$
- the point P has position vector $2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$

The straight line l passes through A and B.

(a) Find a vector equation for l.

(2)

The point C lies on l so that PC is perpendicular to l.

(b) Find the coordinates of C.

(4)

The point P' is the reflection of P in the line l.

(c) Find the coordinates of P'

(2)

(d) Hence find $|\overrightarrow{PP'}|$, giving your answer as a simplified surd.

(2)

(a) Formula for vector line:

$\mathbf{r}$ = position vector + λ(direction vector)

Where the position vector is any point on the line and direction vector is a vector parallel to the line.

Get direction vector:

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= \begin{pmatrix} 5 \\ 6 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 8 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ M1

$\overrightarrow{OA} = \begin{pmatrix} 4 \\ 8 \\ 1 \end{pmatrix}$ (given above)

$\mathbf{r} = \overrightarrow{OA} + \lambda\overrightarrow{AB}$

$\therefore \mathbf{r} = \begin{pmatrix} 4 \\ 8 \\ 1 \end{pmatrix} + \lambda\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ line l

A1

Question 4 continued

(b) Point C is the general point on l, $\therefore \vec{OC} = \begin{pmatrix}4+\lambda\\8-2\lambda\\1+2\lambda\end{pmatrix}$

Hence $\vec{PC} = \vec{OC} - \vec{OP}$ given

$$= \begin{pmatrix}4+\lambda\\8-2\lambda\\1+2\lambda\end{pmatrix} - \begin{pmatrix}2\\-2\\1\end{pmatrix} = \begin{pmatrix}2+\lambda\\10-2\lambda\\2\lambda\end{pmatrix} = \vec{PC}. \quad \text{M1}$$

We're given that $\vec{PC}$ is perpendicular to l, $\therefore$ perpendicular to $\vec{AB}$. Hence dot product=0

Formula for dot product:

$$\begin{pmatrix}a\\b\\c\end{pmatrix}\cdot\begin{pmatrix}d\\e\\f\end{pmatrix} = ad+be+cf$$

part (a)

$$\vec{PC}\cdot\vec{AB} = 0$$

$$\begin{pmatrix}2+\lambda\\10-2\lambda\\2\lambda\end{pmatrix}\cdot\begin{pmatrix}1\\-2\\2\end{pmatrix} = 0$$

$$(2+\lambda) - 2(10-2\lambda) + 2(2\lambda) = 0$$ Solve for λ

$$9\lambda - 18 = 0 \Rightarrow \lambda = 2 \quad \text{dM1}$$

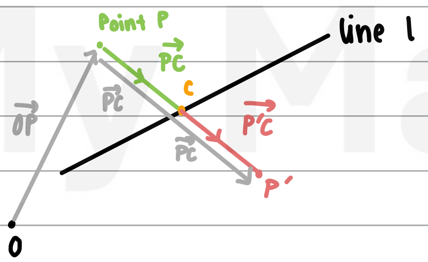
Substitute λ back into $\vec{OC} = \dots$ to get the coordinates.

$$\vec{OC} = \begin{pmatrix}4+2\\8-2(2)\\1+2(2)\end{pmatrix} \quad \text{ddM1}$$

$$= \begin{pmatrix}6\\4\\5\end{pmatrix}$$

$\therefore$ coordinates of C: (6, 4, 5) A1

(b) We need $\vec{OP'}$.



$\vec{OP'}$ is represented by the gray path.

$\therefore \vec{OP'} = \vec{OP} + 2\vec{PC}$ $\quad \vec{PC} = \begin{pmatrix}2+2\\10-2(2)\\2(2)\end{pmatrix} = \begin{pmatrix}4\\6\\4\end{pmatrix}$ from (b), $\vec{PC}$ and $\lambda=2$.

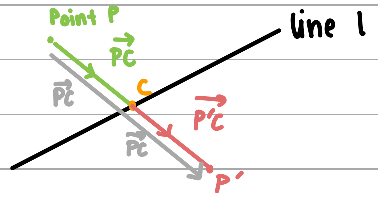
$$= \begin{pmatrix}2\\-2\\1\end{pmatrix} + 2\begin{pmatrix}4\\6\\4\end{pmatrix} \quad \text{M1}$$

$$= \begin{pmatrix}10\\10\\9\end{pmatrix}$$

$\therefore$ coordinates of P': (10, 10, 9) A1

Question 4 continued

(d)



Hence $\vec{PP'} = 2\vec{PC}$ $\therefore$ $|\vec{PP'}| = 2|\vec{PC}|$

Formula for Magnitude of a vector:

Let $\vec{AB} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$.

$$|\vec{AB}| = \sqrt{a^2 + b^2 + c^2}$$

$$\vec{PC} = \begin{pmatrix} 4 \\ 6 \\ 4 \end{pmatrix} \Rightarrow \quad |\vec{PC}| = \sqrt{4^2 + 6^2 + 4^2} \quad \text{M1}$$

$$= 2\sqrt{17}$$

$\therefore$ $|\vec{PP'}| = 2 \times 2\sqrt{17} = 4\sqrt{17}$ $|\vec{PP'}|$ found

A1

Question 4 continued

(Total for Question 4 is 10 marks)

5. (i) Find

$$\int x^2 e^x \, dx$$

(4)

(ii) Use the substitution $u = \sqrt{1-3x}$ to show that

$$\int \frac{27x}{\sqrt{1-3x}} \, dx = -2(1-3x)^{\frac{1}{2}}(Ax+B) + k$$

where A and B are integers to be found and k is an arbitrary constant.

(6)

(i) $\int x^2 e^x \, dx$

multiplication, ∴ use by Parts

★ Integration by Parts:

$\int vu'\,dx = vu - \int uv'\,dx$

OR

SDI table method

Method I : Formula

$\int x^2 e^x \, dx$

$v = x^2 \xrightarrow{\frac{d}{dx}} v' = 2x$

$u = e^x \xleftarrow{\int} u' = e^x$

$= x^2 e^x - 2\int x e^x \, dx$ — multiplication ∴ By Parts again

M1A1

$u = x \xrightarrow{\frac{d}{dx}} u' = 1$

$v = e^x \xleftarrow{\int} v' = e^x$

$= x^2 e^x - 2xe^x + 2\int e^x \, dx$

M1

$= x^2 e^x - 2xe^x + 2e^x + c$ A1

Method II : SDI

$\int x^2 e^x \, dx$:

SDI – table

Sign	Differentiate	Integrate
+	x^2	e^x
−	$2x$	e^x
+	2	e^x
−	0	e^x

$\Rightarrow \int x^2 e^x \, dx = x^2 e^x - 2xe^x + 2e^x + c$ A1

M1A1M1

P 7 4 3 2 9 A 0 1 8 3 2

Question 5 continued

(ii) To use the given substitution $u=\sqrt{1-3x}$, we need to find $dx = \ldots$ and $x = \ldots$

To get dx, differentiate $u=(1-3x)^{\frac{1}{2}}$:

$$u=(1-3x)^{\frac{1}{2}}$$

$$\frac{du}{dx} = \frac{1}{2}(1-3x)^{-\frac{1}{2}} \cdot -3$$ chain rule: multiply by the derivative of the bracket.

$$\frac{du}{dx} = -\frac{3}{2}(1-3x)^{-\frac{1}{2}}$$

Also, use $u=\sqrt{1-3x} \Rightarrow u^2 = 1-3x$ to make it easier:

$$\frac{du}{dx} = -\frac{3}{2}\underbrace{(1-3x)}_{=u^2}^{-\frac{1}{2}}$$

$$\therefore \frac{du}{dx} = -\frac{3}{2}(u^{\cancel{2}})^{\cancel{-\frac{1}{2}}\,-1}$$

$$\frac{du}{dx} = -\frac{3}{2u}$$ B1

$$dx = -\frac{2u}{3}du$$

To get $x = \ldots$, rearrange:

$$u=\sqrt{1-3x}$$

$$u^2 = 1-3x$$

$$3x = 1-u^2$$

$$x = \frac{1-u^2}{3}$$

Substitute into the Integration:

$$\int \frac{27x}{\sqrt{1-3x}}\,dx$$ u

$$= \int \frac{27\left(\frac{1-u^2}{3}\right)}{u}\left(-\frac{2u}{3}du\right)$$ M1

$$= 27\int \frac{\frac{1-u^2}{3}}{\cancel{u}} \cdot -\frac{2\cancel{u}}{3}\,du$$

$$= 27\int -\frac{2(1-u^2)}{9}\,du$$

$$= -\frac{2}{9}\cdot 27\int 1-u^2\,du$$

$$= -6\int 1-u^2\,du$$

A1 $$= 6\int u^2-1\,du$$ we can easily integrate this

Question 5 continued

(ii) cont.

$$= 6\int u^2 - 1 \, du$$

$$= 6\left(\tfrac{1}{3}u^3 - u\right) \quad \text{A1}$$

Substitute $u = \sqrt{1-3x}$ back in:

$$= 6\left(\tfrac{1}{3}\left(\sqrt{1-3x}\right)^3 - \sqrt{1-3x}\right) \quad \text{M1}$$

$$= 2(1-3x)^{\frac{3}{2}} - 6(1-3x)^{\frac{1}{2}}$$

$$= 2(1-3x)^{\frac{1}{2}}\left((1-3x) - 3\right)$$ factor out $2(1-3x)^{\frac{1}{2}}$

$$= -2(1-3x)^{\frac{1}{2}}(2+3x) + k$$ hence shown

A1

Question 5 continued

(Total for Question 5 is 10 marks)

6. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The temperature, θ °C, of a car engine, t minutes after the engine is turned off, is modelled by the differential equation

$$\frac{d\theta}{dt} = -k(\theta - 15)^2$$

where k is a constant.

Given that the temperature of the car engine
- is 85 °C at the instant the engine is turned off
- is 40 °C exactly 10 minutes after the engine is turned off

(a) solve the differential equation to show that, according to the model

$$\theta = \frac{at + b}{ct + d}$$

where a, b, c and d are integers to be found.

(7)

(b) Hence find, according to the model, the time taken for the temperature of the car engine to reach 20 °C. Give your answer to the nearest minute.

(2)

(a) $\frac{d\theta}{dt} = -k(\theta - 15)^2$

$\frac{1}{(\theta-15)^2} \, d\theta = -k \, dt$ Separate Variables

$\int (\theta-15)^{-2} \, d\theta = \int -k \, dt$ Integrate

Multiply both sides by -1 B1

$-(\theta-15)^{-1} = -kt + c$ M1A1

We are given that at $t = 0, \theta = 85$:

$-(85-15)^{-1} = c$

$-\frac{1}{70} = c$ value of c

M1

Also that $t = 10, \theta = 40$:

$-(40-15)^{-1} = -k(10) - \frac{1}{70}$

$-\frac{1}{25} = -10k - \frac{1}{70}$

M1 $k = \frac{9}{3500}$ value of k

P 7 4 3 2 9 A 0 2 2 3 2

Question 6 continued

(a) cont.

Substitute $c = -\frac{1}{70}$ and $k = \frac{9}{3500}$ into the integrated equation:

$$\frac{1}{\theta - 15} = \frac{9}{3500}t + \frac{1}{70}$$

Make θ the subject:

$$1 = \frac{9}{3500}t(\theta - 15) + \frac{1}{70}(\theta - 15)$$ multiply by $(\theta - 15)$

$$3500 = 9t(\theta - 15) + 50(\theta - 15)$$ multiply by 3500

$$3500 = 9t\theta - 135t + 50\theta - 750$$ expand

$$9t\theta + 50\theta = 135t + 4250$$ collect all θ terms on 1 side

$$\theta(9t + 50) = 135t + 4250$$ M1 factor θ out

$$\theta = \frac{135t + 4250}{9t + 50}$$ divide by $(9t + 50)$

A1

(b) $\theta = 20^\circ$

$$20 = \frac{135t + 4250}{9t + 50}$$ Solve for t

M1

$$180t + 1000 = 135t + 4250$$

$$45t = 3250$$

$$t = 72.2\dot{2}$$

$$\therefore t = 72 \text{ minutes}$$ A1

Question 6 continued

Question 6 continued

(Total for Question 6 is 9 marks)

7. Use proof by contradiction to prove that $\sqrt{7}$ is irrational.

(You may assume that if k is an integer and k^2 is a multiple of 7 then k is a multiple of 7)

(4)

Make an Assumption:

"Assume that $\sqrt{7}$ is a rational number"

M1

We want our assumption to be the opposite of what we are trying to prove.

In the Proof, we are trying to contradict the assumption

Since $\sqrt{7}$ is rational, it can be written as:

$\sqrt{7} = \frac{a}{b}$ where a and b have no common factors

$7 = \frac{a^2}{b^2} \Rightarrow 7b^2 = a^2$

Hence a^2 is a multiple of 7, and $\therefore$ a is also a multiple of 7. A1

a can hence be written as: $a = 7k$

$7 = \frac{(7k)^2}{b^2}$

$7b^2 = 49k^2$

$b^2 = 7k^2$ M1

Hence b^2 is a multiple of 7, and $\therefore$ b is also a multiple of 7.

So both a and b are multiples of 7, which contradicts the fact that a and b have no factors in common. Hence it is proven by contradiction that $\sqrt{7}$ is irrational.

A1

Question 7 continued

(Total for Question 7 is 4 marks)

8.

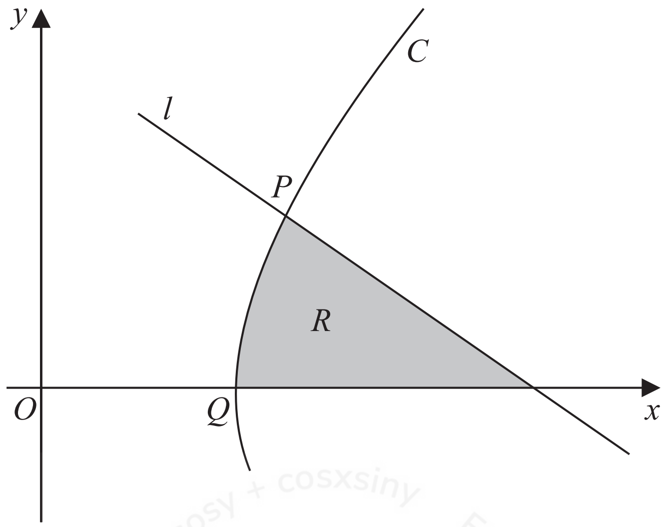


Figure 2

Figure 2 shows a sketch of part of the curve C with parametric equations

$$x = t + \frac{1}{t} \qquad y = t - \frac{1}{t} \qquad t > 0.7$$

The curve C intersects the x-axis at the point Q.

(a) Find the x coordinate of Q.

(1)

The line l is the normal to C at the point P as shown in Figure 2.

Given that $t = 2$ at P

(b) write down the coordinates of P

(1)

(c) Using calculus, show that an equation of l is

$$3x + 5y = 15$$

(3)

The region, R, shown shaded in Figure 2 is bounded by the curve C, the line l and the x-axis.

(d) Using algebraic integration, find the exact volume of the solid of revolution formed when the region R is rotated through 2π radians about the x-axis.

(7)

P 7 4 3 2 9 A 0 2 8 3 2

DO NOT WRITE IN THIS AREA

Question 8 continued

(a) At x-axis $y=0$, use this to get value of t:

$$0 = t - \frac{1}{t}$$

$$t^2 = 1$$

$$t = \pm 1 \rightarrow t = 1 \text{ since } t > 0.7$$

Substitute into t into $x = \dots$:

$$x = 1 + \frac{1}{1} \Rightarrow x = 2$$ B1

(b) Given $t=2$ at P:

$x = 2 + \frac{1}{2} = 2.5$

$y = 2 - \frac{1}{2} = 1.5$

$\therefore P(2.5, 1.5)$ B1

(c) ★ Parametric differentiation:

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$y = t - \frac{1}{t}$ | $x = 1 + \frac{1}{t}$

$\frac{dy}{dt} = 1 + \frac{1}{t^2}$ | $\frac{dx}{dt} = 1 - \frac{1}{t^2}$

Substitute:

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \Rightarrow \frac{1 + \frac{1}{t^2}}{1 - \frac{1}{t^2}} = \frac{dy}{dx}$$ M1

At P $t=2$, $\therefore$ gradient of tangent:

$$m = \frac{dy}{dx} = \frac{1 + \frac{1}{4}}{1 - \frac{1}{4}} = \frac{\frac{5}{4}}{\frac{3}{4}} = \frac{5}{3}$$ tangent

gradient of normal: $m(\text{normal}) = -\frac{1}{m(\text{tangent})}$

$\therefore m(\text{normal}) = -\frac{3}{5}$

Use $y - y_1 = m(x - x_1)$ to get the equation of the line

$m = -\frac{3}{5}$

$y_1 = 1.5$

$x_1 = 2.5$

$$y - 1.5 = -\frac{3}{5}(x - 2.5)$$ dM1

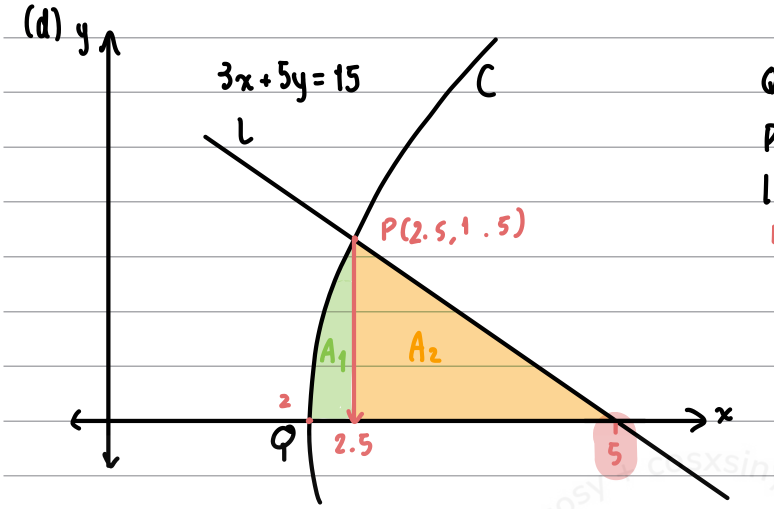
$$5y - \frac{15}{2} = -3x + \frac{15}{2}$$

$$3x + 5y = 15$$ hence shown

A1

Question 8 continued

(d)



$Q_x = 2$

$P_x = 1.5$

$l_x = 5$

L $3x + 5(0) = 15$

$3x = 15$ $x = 5$ B1

$A_1 \longrightarrow V_1$ (volume)

$A_2 \longrightarrow A_2$ (volume)

Area $A_1 \longrightarrow$ Parametric Integration, Volumes of Revolution

Formula for Volume of Revolution around the x-axis:

$$V = \pi\int_a^b y^2\,dx \;=\; \text{Parametric} \Rightarrow V = \pi\int y^2 \frac{dx}{dt}\,dt$$

Limits in t:

$x = 2 \rightarrow t = 1$

$x = 2.5 \rightarrow t = 2$

$$V_1 = \pi\int_1^2 \left(t - \frac{1}{t}\right)^2 \cdot \left(1 - \frac{1}{t^2}\right) dt \quad \text{M1}$$

$$= \pi\int_1^2 \left(t^2 - 2 + \frac{1}{t^2}\right) \cdot \left(1 - \frac{1}{t^2}\right) dt \quad \text{expand}$$

$$= \pi\int_1^2 t^2 - 1 - 2 + \frac{2}{t^2} + \frac{1}{t^2} - \frac{1}{t^4}\,dt$$

$$= \pi\int_1^2 t^2 + \frac{3}{t^2} - \frac{1}{t^4} - 3\,dt \quad \text{M1}$$

$$= \pi\left[\frac{1}{3}t^3 - \frac{3}{t} + \frac{1}{3t^3} - 3t\right]_1^2 \quad \text{A1}$$

$$= \pi\left(\left(\frac{1}{3}(2)^3 - \frac{3}{2} + \frac{1}{3(2)^3} - 3(2)\right) - \left(\frac{1}{3}(1)^3 - \frac{3}{1} + \frac{1}{3(1)^2} - 3(1)\right)\right)$$

$$= \pi\left(\frac{8}{3} - \frac{3}{2} + \frac{1}{24} - 6 - \frac{1}{3} + 3 - \frac{1}{3} + 3\right) \quad \text{M1}$$

$$V_1 = \frac{13}{24}\pi$$

Question 8 continued

$A_2 \longrightarrow$ Volume$_2$, V_2, it's a cone

Volume of a cone:

$V = \pi r^2 \frac{h}{3}$

$r = 1.5$ (P_y)

$h = 5 - 2.5 = 2.5$ $(l_x - P_x)$

$V_2 = \pi \cdot (1.5)^2 \cdot \frac{2.5}{3}$

$= \pi \cdot \frac{9}{4} \cdot \frac{5}{6}$

$= \frac{45}{24}\pi$

$V_2 = \frac{15}{8}\pi$ M1

Volume $R = V_1 + V_2$

$= \frac{13}{24}\pi + \frac{15}{8}\pi$

$= \frac{29}{12}\pi$ Volume R

A1

Question 8 continued

(Total for Question 8 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS